THE EXAMPLES OF USING THE (DIRECT) COMPARISON TEST PRESENTED IN CLASS.

EXAMPLE 1: Is the series $\sum_{n=1}^{\infty} \frac{1}{n^2+6} = \sum_{n=1}^{\infty} c_n$ C OR D?

SOL'N: $c_n = \frac{1}{n^2+6}$ Looks Like $\frac{1}{n^2}$

and $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

A FIRST JUSTIFICATION IS REQUIRED.

"The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with $p = 2 > 1$,

so $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is Convergent."

Since $n^2+6 \geq n^2$, $\frac{1}{n^2+6} \leq \frac{1}{n^2}$ and $c_n \leq b_n$, $n \geq 1$

A SECOND JUSTIFICATION IS REQUIRED.

"BECAUSE ① $\frac{1}{n^2+6} \leq \frac{1}{n^2}$ for all $n \geq 1$

and ② the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is Convergent,

the series $\sum_{n=1}^{\infty} \frac{1}{n^2+6}$ is Convergent by

the (DIRECT) Comparism TEST."

(Using the Direct Comparison Test)

EXAMPLE 2: Is the series $\sum_{n=1}^{\infty} \frac{1}{n+6}$ C or D?

Sol'n: Since $n+6 > n$, $\frac{1}{n+6} < \frac{1}{n}$ and $\sum \frac{1}{n}$ is D,

BUT, these facts give us no information about C or D for the smaller Dominated series.

We need a series $\sum_{n=1}^{\infty} a_n$ with $a_n < \frac{1}{n+6}$ such that $\sum a_n$ is Divergent. The series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{2n}$ works like this!

When $n \geq 6$, $n+n = 2n \geq n+6$, so $\frac{1}{2n} \leq \frac{1}{n+6}$.

THE FIRST JUSTIFICATION

"The series $\sum_{n=1}^{\infty} \frac{1}{2n} = \sum_{n=1}^{\infty} \frac{1}{2} \cdot \frac{1}{n}$ is a term-by-term constant multiple of the harmonic series, so $\sum_{n=1}^{\infty} \frac{1}{2n}$ is Divergent, since $\sum_{n=1}^{\infty} \frac{1}{n}$ is Divergent".

THE SECOND JUSTIFICATION.

"Because ① $\frac{1}{2n} \leq \frac{1}{n+6}$ for all $n \geq 6$,

and ② $\sum_{n=1}^{\infty} \frac{1}{2n}$ is Divergent,

The series $\sum_{n=1}^{\infty} \frac{1}{n+6}$ is Divergent by the (Direct) Comparison Test."